Calculus II .. MATH 1080
Integration by Parts

Evaluate the Integral:

$$\int_0^{\pi/20} w^2 \sin(10w)\,dw$$

$du = 2w \quad \int_0^{\pi/20} u\sin(5u)\,du$

Step 1: Think about techniques.

~~* U-sub~~

x anti-derivatives $\quad \int 2x\,dx = x^2$

* trig functions

X simplify the integrand

* similar techniques

* integration by parts

$$\int w^2 \sin(10w)\,dw$$

gration by parts

(a) STEP 1

write out the formula

$$\int u\,dv = uv - \int v\,du$$

$$\int u\,dv$$

appears to be the product of 2 functions

$u = g(x) \quad dv = f'(x)$.

(b) STEP 2

Substitute your given equation into the formula

* this is 2 parts: ① assign u, v, du, dv ② plug into formula.

$$\int w^2 \sin(10w)\,dw$$

$u = w^2 \qquad dv = \sin(10w)\,dw$

$du = 2w\,dw \qquad v = -1/10 \cos(10w)$

$$\int w^2 \sin(10w)\,dw = -\frac{w^2}{10}\cos(10w) - \int -\frac{1}{5} w\cos(10w)\,dw$$

Integration by parts

(c) STEP 3

$$\int w^2 \sin(10w)\,dw = -\frac{w^2}{10}\cos(10w) - \underbrace{\int -\frac{1}{5} w\cos(10w)\,dw}_{\text{integrate}}$$

looks just like the original

$$-\int -\frac{1}{5} w\cos(10w)\,dw$$

$x = w$ $\quad dy = \cos(10w)\,dw$

$dx = 1\,dw$ $\quad y = \frac{1}{10}\sin(10w)$

$$= -\left(-\frac{1}{5}\right)\int w\cos(10w)\,dw \qquad x, y, dx, dy$$

$$= \frac{1}{5}\int w\cos(10w)\,dw = \frac{1}{5}\left(\frac{w}{10}\sin(10w) - \int \frac{1}{10}\sin(10w)\,dw\right)$$

$$\int w^2\sin(10w)\,dw = -\frac{w^2}{10}\cos(10w) + \frac{1}{5}\left(\frac{w}{10}\sin(10w) - \int \frac{1}{10}\sin(10w)\,dw\right)$$

(d) STEP 4 (sometimes step)

$$\int \frac{1}{10}\sin(10w)\,dw.$$

$$-\frac{\cos(10w)}{100}$$

$$\int w^2\sin(10w)\,dw = -\frac{w^2}{10}\cos(10w) + \frac{1}{50}w\sin(10w) + \frac{1}{500}\cos(10w) + C$$

Indefinite integral.

(e) STEP 5 (sometimes) Evaluate at bounds

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du.$$

Integration by Parts

(c) Step 5 (sometimes) Evaluate at bounds.

$$\int_0^{\pi/20} w^2 \sin(10w)\,dw = \left[-\frac{w^2}{10}\cos(10w) + \frac{w}{50}\sin(10w) + \frac{1}{500}\cos(10w)\right]\Bigg|_0^{\pi/20}$$

$$\left[-\frac{(\pi/20)^2}{10}\cos\left(\frac{\pi}{20}\cdot 10\right) + \frac{\pi/20}{50}\sin\left(\frac{\pi}{20}\cdot 10\right) + \frac{1}{500}\left(\cos\left(10\cdot \pi/20\right)\right)\right] -$$

$$\left[\frac{(0)^2}{10}\cos(10\cdot 0) + \frac{0}{50}\sin(0\cdot 10) + \frac{1}{500}\cos(10\cdot 0)\right]$$

$$\left[-\frac{(\pi/20)^2}{10}\cdot \cancel{\cos}\left(\frac{\pi}{2}\right)^{0} + \frac{\pi/20}{50}\cancel{\sin}\left(\frac{\pi}{2}\right)^{1} + \frac{1}{\cancel{500}}\cos\left(\frac{\pi}{2}\right)^{0}\right]$$

$$\frac{\pi}{20}\cdot\frac{1}{50} = \frac{\pi}{1000}$$

$$\cancel{0\cdot\cos(0)}^{1} + \cancel{0\cdot\sin(0)}^{0} + \frac{1}{500}\cancel{\cos(0)}^{1} = \frac{1}{500}$$

$$\frac{\pi}{1000} - \frac{1}{500} = \frac{\pi}{1000} - \frac{2}{1000} = \frac{\pi-2}{1000} \approx 0.00114159.$$

$$\frac{1}{500}\left(\frac{2}{2}\right) = \frac{2}{1000}$$

$$\int u\,dv = uv - \int v\,du.$$