

Calc 1080

Telescoping Series

Determine if converges or diverges. If converges find value.

$$\sum_{n=0}^{\infty} \frac{1}{n^2+3n+2}$$

$$S_n = \sum_{j=0}^n \frac{1}{j^2+3j+2}$$

$$\frac{1}{i^2+3i+2} = \frac{1}{(i+2)(i+1)} = \frac{1}{i+1} - \frac{1}{i+2}$$

$$S_n = \sum_{i=0}^n \left(\frac{1}{i+1} - \frac{1}{i+2} \right)$$

$$= \left(1 - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \dots + \left(\frac{1}{n} - \frac{1}{n+1} \right) + \left(\frac{1}{n+1} - \frac{1}{n+2} \right)$$
$$= \underline{1 - \frac{1}{n+2}}$$

$$\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n+2} \right) = 1$$

$$\sum_{n=0}^{\infty} \frac{1}{n^2+3n+2} = 1$$