Method of Integrating Factor:

$$\frac{1}{x}\frac{dy}{dx} + \frac{3y}{x^2} + \frac{2}{x} = 3$$

Step 1 Standard form: $\frac{dy}{dx} + P(x)y = Q(x)$

$$\frac{dy}{dx} + \frac{3y}{x} + 2 = 3x \Rightarrow \frac{dy}{dx} + \frac{3y}{x} = 3x - 2$$

Step 2 $P(x) = \frac{3}{x}$

Step 3 $\mu(x) = e^{\int P(x)\,dx}$

$$\int P(x)\,dx = \int \frac{3}{x}\,dx = 3\int \frac{1}{x}\,dx = 3\ln|x|$$

$$e^{3\ln|x|} = e^{\ln(x^3)} = x^3$$

Step 4 $\frac{dy}{dx}\cdot x^3 + \frac{3y}{x}\cdot x^3 = (3x-2)x^3$

$$\rightarrow \underbrace{\frac{dy}{dx}\cdot x^3 + 3x^2y} = 3x^4 - 2x^3$$

$$\rightarrow \frac{d}{dx}[x^3y]$$

check: product rule

$$\frac{d}{dx}[x^3y] = \frac{d}{dx}[x^3]\cdot y + \frac{d}{dx}[y]\cdot x^3$$

$$3x^2y + \frac{dy}{dx}\cdot x^3 \quad ✓$$

$\rightarrow$ * LHS $= \frac{d}{dx}[\mu(x)\cdot y]$ *

Step 5 Substitute for LHS and & solve:

$$\frac{d}{dx}[x^3y] = 3x^4 - 2x^3$$

$$\frac{dy}{dx}[x^3] = 3x^4 - 2x^3$$

$$\int x^3\,dy = \int (3x^4 - 2x^3)\,dx$$

$$x^3y = \frac{3}{5}x^5 - \frac{2}{4}x^4 + C$$

$$\boxed{y = \frac{3}{5}x^2 - \frac{1}{2}x + C}$$

Method of Integrating Factor

Step 1: write the diff. equation in standard form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Step 2: Identify $P(x)$

Step 3: Find the integrating factor, $\mu(x)$

$$\mu(x) = e^{\int P(x)\,dx}$$

Step 4: multiply by $\mu(x)$ on both sides of the standard form differential equation.

Step 5: Substitute for the LHS

$$\frac{d}{dx}[\mu(x)\cdot y]$$

Solve the equation