

## Calculus 2080 Mixing Problem Example

A 1500 gallon tank initially contains 600 gallons of water with 5 lbs of salt dissolved. Water enters into the tank at a rate of 9 gal/hr and the water has a concentration of  $\frac{1}{5}(1 + \cos(t))$  lbs/gal. If a well-mixed solution leaves the tank at a rate of 6 gal/hr, how much salt is in the tank when it overflows?

$Q(t)$  → amt of salt in tank @ time  $t$

$$\text{concentration} = \frac{Q(t)}{V(t)} \quad V(t) = 600 + 3t$$

$$Q'(t) = 9 \left( \frac{1}{5}(1 + \cos(t)) \right) - 6 \frac{Q(t)}{600 + 3t} \quad Q(0) = 5$$

$$= \frac{9}{5}(1 + \cos(t)) - \frac{2Q(t)}{200 + t}$$

$$Q'(t) + \frac{2Q(t)}{200 + t} = \frac{9}{5}(1 + \cos(t)) \quad \mu(t) = e^{\int \frac{2}{200+t} dt} = e^{2 \ln(200+t)} = (200+t)^2$$

$$\int ((200+t)^2 Q(t))' dt = \int \frac{9}{5} (200+t)^2 (1 + \cos(t)) dt$$

$$(200+t)^2 Q(t) = \frac{9}{5} \left[ \frac{1}{3} (200+t)^3 + (200+t)^2 \sin(t) + 2(200+t) \cos(t) - 2 \sin(t) \right] + C$$

$$Q(t) = \frac{9}{5} \left[ \frac{1}{3} (200+t) + \frac{\sin(t)}{200+t} + \frac{2 \cos(t)}{(200+t)^2} - \frac{2 \sin(t)}{(200+t)^2} \right] + \frac{C}{(200+t)^2}$$

$$\text{IC: } Q(0) = 5 = \frac{9}{5} \left( \frac{1}{3} (200) + \frac{2}{200} \right) + \frac{C}{200^2} \rightarrow C = -4600720$$

$$Q(t) = \frac{9}{5} \left( \frac{1}{3} (200+t) + \frac{\sin(t)}{200+t} + \frac{2 \cos(t)}{(200+t)^2} - \frac{2 \sin(t)}{(200+t)^2} \right) - \frac{4600720}{(200+t)^2}$$

$$V(t) = 1500 = 600 + 3t$$

$$t = 300 \text{ hr}$$

$$Q(300) = 279.797 \text{ lbs}$$