Content Embedded Inquiry: Preparing Mathematics Teachers for the New Standards

Dr. Bob Horton
Dr. Andy Tyminski
Clemson University
Eugene T. Moore School of Education

Common Core State Standards for Mathematics (CCSSM)

- The CCSSM outline **what** mathematics students should learn at each grade level (Content) and just as importantly, **how** they should learn it (Mathematical Practices).
- These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step.
- Our challenge is to find a way to organize mathematics instruction such that students can engage in the Mathematical Practices while simultaneously developing understanding of significant mathematical content. Content-embedded inquiry is one way to accomplish this goal.

What content is appropriate for inquiry? Standards for Mathematical Content

- The Standards for Mathematical Content are a balanced combination of procedure and understanding. **Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content**. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices** (CCSSM, 2011, p. 8)

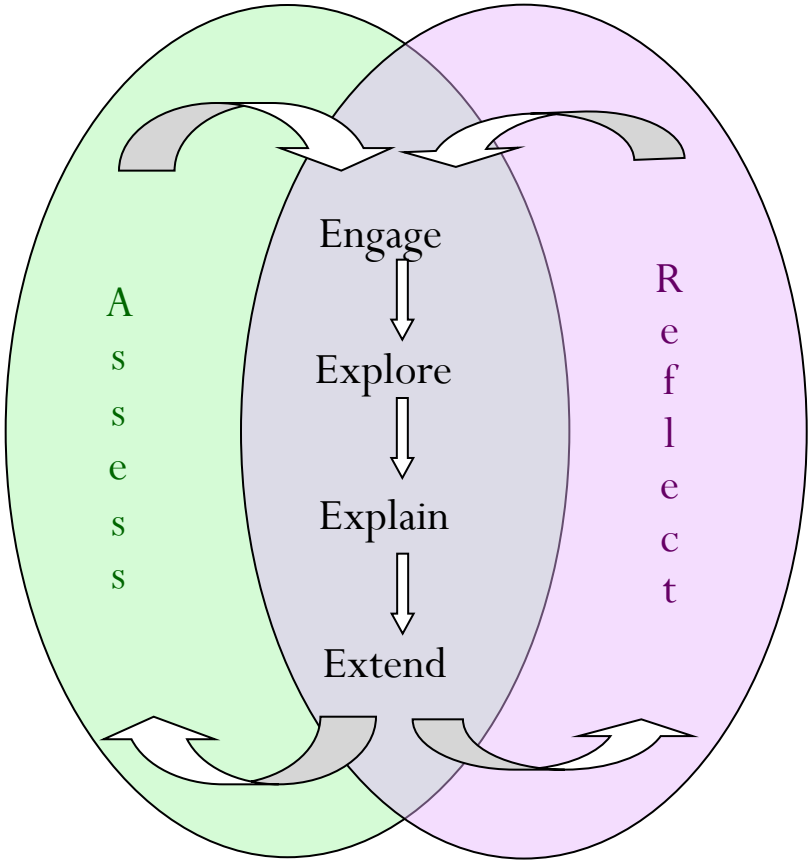
What does inquiry look like? Standards of Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

One approach to content based inquiry

- Identify the big ideas in the grade level standards; focus on standards designed to introduce or develop understanding of a mathematical concept.
- Consider prior knowledge and misconceptions students might hold concerning this concept. How can you uncover these?
- Select rich mathematical tasks that allow students to engage in development of new ideas by building on their **current** conceptual and procedural knowledge.
- Organize and orchestrate a mathematical discussion built from students' mathematical thinking that elicits and illuminates the intended mathematical ideas.

4E x 2 INSTRUCTIONAL MODEL



See www.clemson.edu/iim for more details.

4E X 2

- **Engage** – activity/task designed to introduce the topic at hand. Should be used to probe for misconceptions and activate prior knowledge as well as motivate students.
- **Explore** – activity/tasks designed to allow students to think/reason mathematically on a specific topic. The teacher is in the role of supporting this thinking.
- **Explain** – the portion of the lesson where students explain their thinking. The teacher organizes and connects this discussion to more formalized mathematical ideas and terminology.
- **Extend** – other "Explore" activity/tasks designed to build on previous mathematical knowledge. May involve application, generalizations or proof.

Ratio and Proportion

Mixing Juice

Adapted from Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2009). *Comparing and scaling: Ratio, proportion, and percent*. Upper Saddle River, NJ: Pearson.

6.RP.1, 6.RP.2, 6.RP3

7.RP.1, 7.RP.2

But first, a question…

- Sam the snake is 4 feet long. When he is fully grown, he will be 8 feet long. Sally the snake is 5 feet long. When she is fully grown, she will be 9 feet long. Which snake is closer to being fully grown? Explain how you know. (Thompson & Bush, 2003)

Mixing Juice

- Have you ever made orange juice from frozen orange concentrate? Describe what you had to do.

Mixing Juice

- Bob and Andy attend summer camp. Everyone at the camp helps with the cooking and cleanup at meal times.
- One morning, Bob and Andy make orange juice for all the campers. They plan to make the juice by mixing water and frozen orange-juice concentrate. To find the mix that tastes best they decide to test some mixes.

MIX A	
2 cups concentrate	3 cups cold water

MIX B	
1 cups concentrate	4 cups cold water

MIX C	
2 cups concentrate	4 cups cold water

MIX D	
3 cups concentrate	5 cups cold water

Mixing Juice

1. Which mix do you predict will make juice that is the most "orangey"?
2. Which mix do you predict will make juice that is the least "orangey"?

MIX A	
2 cups concentrate	3 cups cold water

MIX B	
1 cups concentrate	4 cups cold water

MIX C	
2 cups concentrate	4 cups cold water

MIX D	
3 cups concentrate	5 cups cold water

Mixing Juice

- Separate students into four groups. Each group will make a batch of one juice mixture. After, the students taste each to see which tastes the most and least orangey.

MIX A	
2 cups concentrate	3 cups cold water

MIX B	
1 cups concentrate	4 cups cold water

MIX C	
2 cups concentrate	4 cups cold water

MIX D	
3 cups concentrate	5 cups cold water

Mixing Juice

- Now that our mixtures have all made made, we need to taste each and decide which has the most orangey flavor and which has the least orangey flavor.
  - Remember – we are **not** choosing which mix we "like the best."

- Class vote on most and least orangey mixes.

- Based on our discussion, we think:
  - Mix __ has the most orangey flavor
  - Mix __ has the least orangey flavor

Mixing Juice

1. Our next task is to explain **mathematically** which mix will make juice that is the most "orangey." Explain mathematically which mix will make juice that is the least "orangey." Find as many different ways as you can to answer these questions.

MIX A	
2 cups concentrate	3 cups cold water

MIX B	
1 cups concentrate	4 cups cold water

MIX C	
2 cups concentrate	4 cups cold water

MIX D	
3 cups concentrate	5 cups cold water

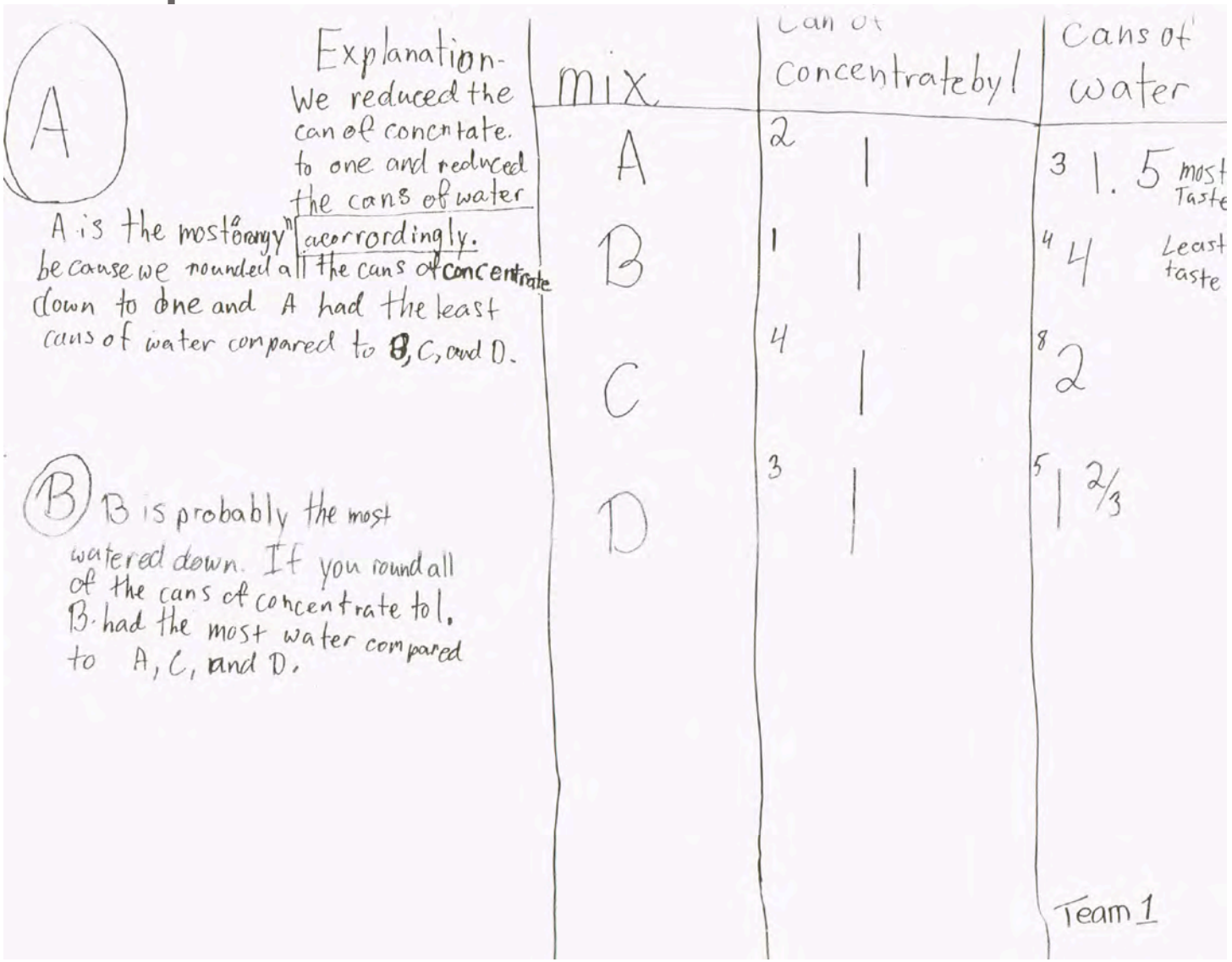
Mixing Juice

- We will work using "Think-Pair-Share". You will be given time to work on the problem individually and will be told when you can talk quietly with your neighbor or group.
- Your small group will be asked to come to consensus and create a group solution – write solution on chart paper
  - Expectations for group work – everyone must understand and be able to explain group's ideas. Dr. T may call on random group members to answer questions

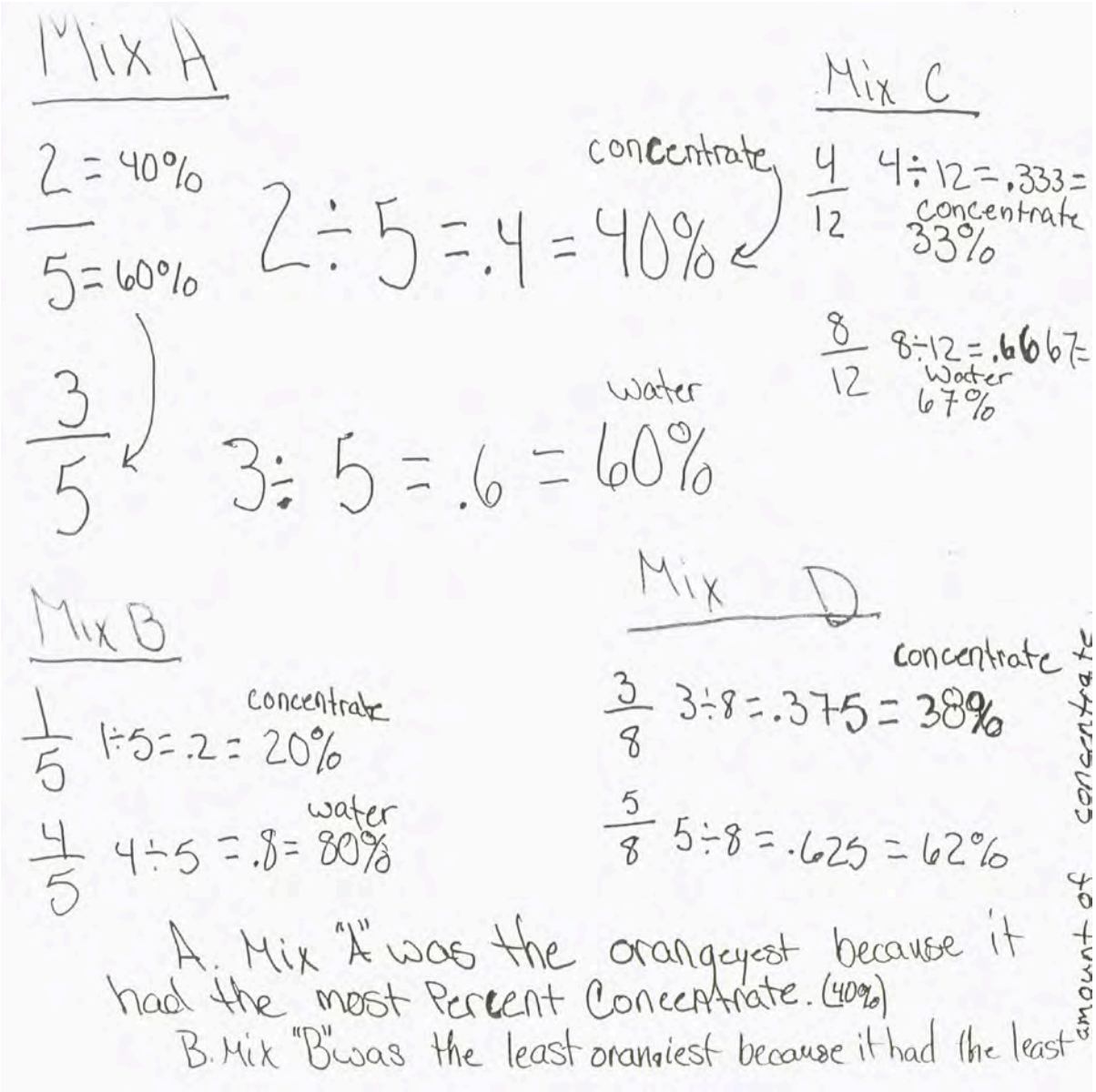
Mixing Juice

- Sharing solutions…

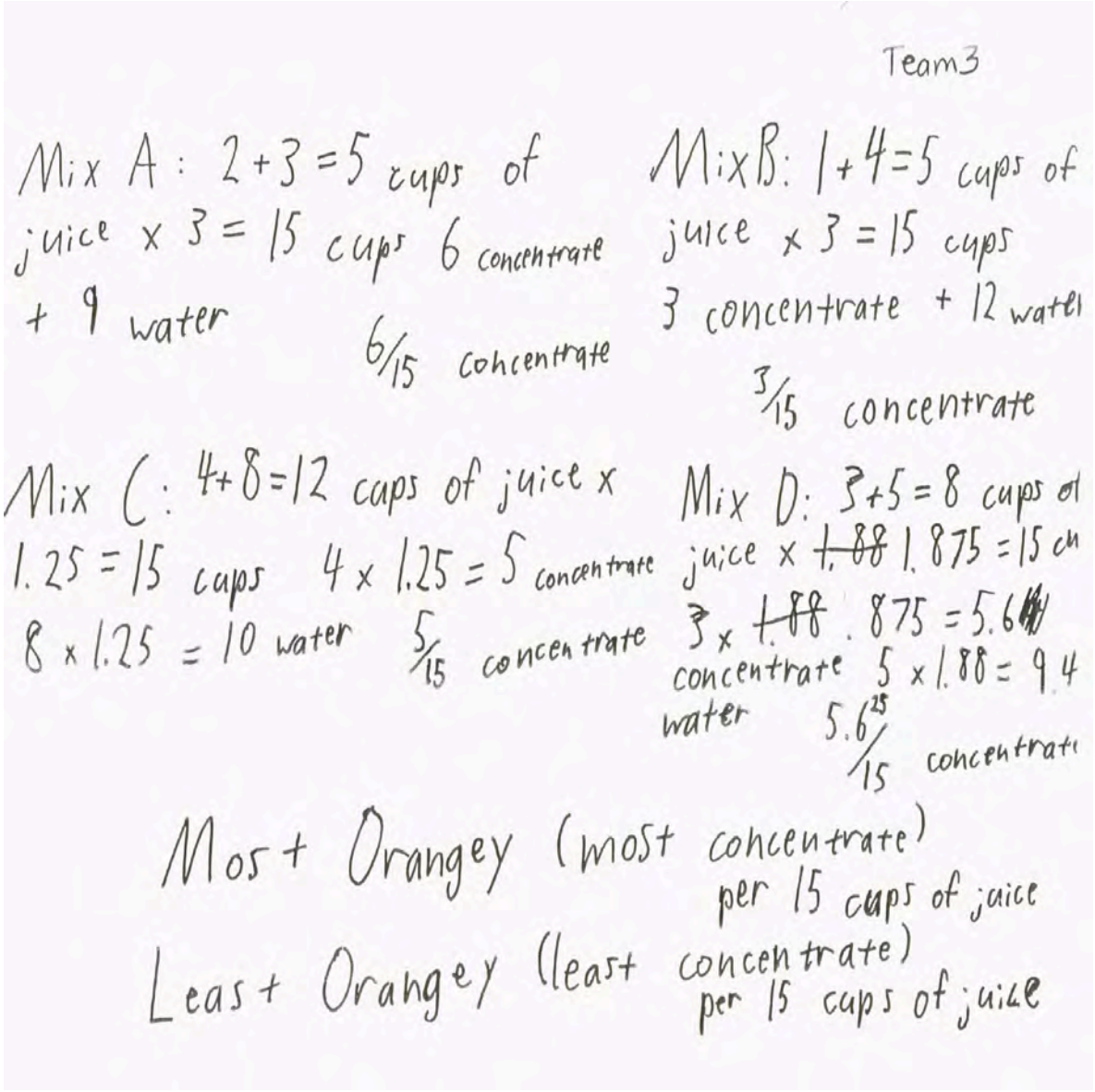
Sample Solutions



Other Solutions



Other Solutions



Team3

Mix A: 2+3=5 cups of juice x 3 = 15 cups 6 concentrate + 9 water

6/15 concentrate

Mix B: 1+4=5 cups of juice x 3 = 15 cups

3 concentrate + 12 water

3/15 concentrate

Mix C: 4+8=12 cups of juice x 1.25 = 15 cups 4 x 1.25 = 5 concentrate

8 x 1.25 = 10 water 5/15 concentrate

Mix D: 3+5 = 8 cups of juice x ~~1.88~~ 1.875 = 15 cups

~~3~~ x ~~1.88~~ .875 = 5.6

concentrate 5 x 1.88 = 9.4

water 5.6 25/15 concentrate

Most Orangey (most concentrate) per 15 cups of juice

Least Orangey (least concentrate) per 15 cups of juice

Other Solutions

A. mix "A" is the most oarngyest because it has the least amount of water added which is $1\frac{1}{2}$ cups of water.

B. mix "B" is the least frutyest because it has 4 cups of water to 1 cup of concentrate.

Mix A

$\frac{3}{2} = \frac{1}{1\frac{1}{2}}$ con. / water

Mix B

$\frac{1}{4}$ con. / water

Mix C

$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$ con. / water

Mix D

$\frac{3}{5} = \frac{1}{1\frac{2}{3}}$ con. / water

Other Solutions

Team 10

Mix A is most orangy, because...

100% ÷ 5 = 20% 1 cup = 20% of the mixture

20% × 2 cups = 40% concentrate
Concentrate

Mix B is the least orangy because...

100% ÷ 5 = 20% 1 cup = 20%
(1+4=5)

1 cup × 20% = 20% concentrate

Mix C is neither because...

100% ÷ 12 = 8% 1 cup = 8%
(4+8)

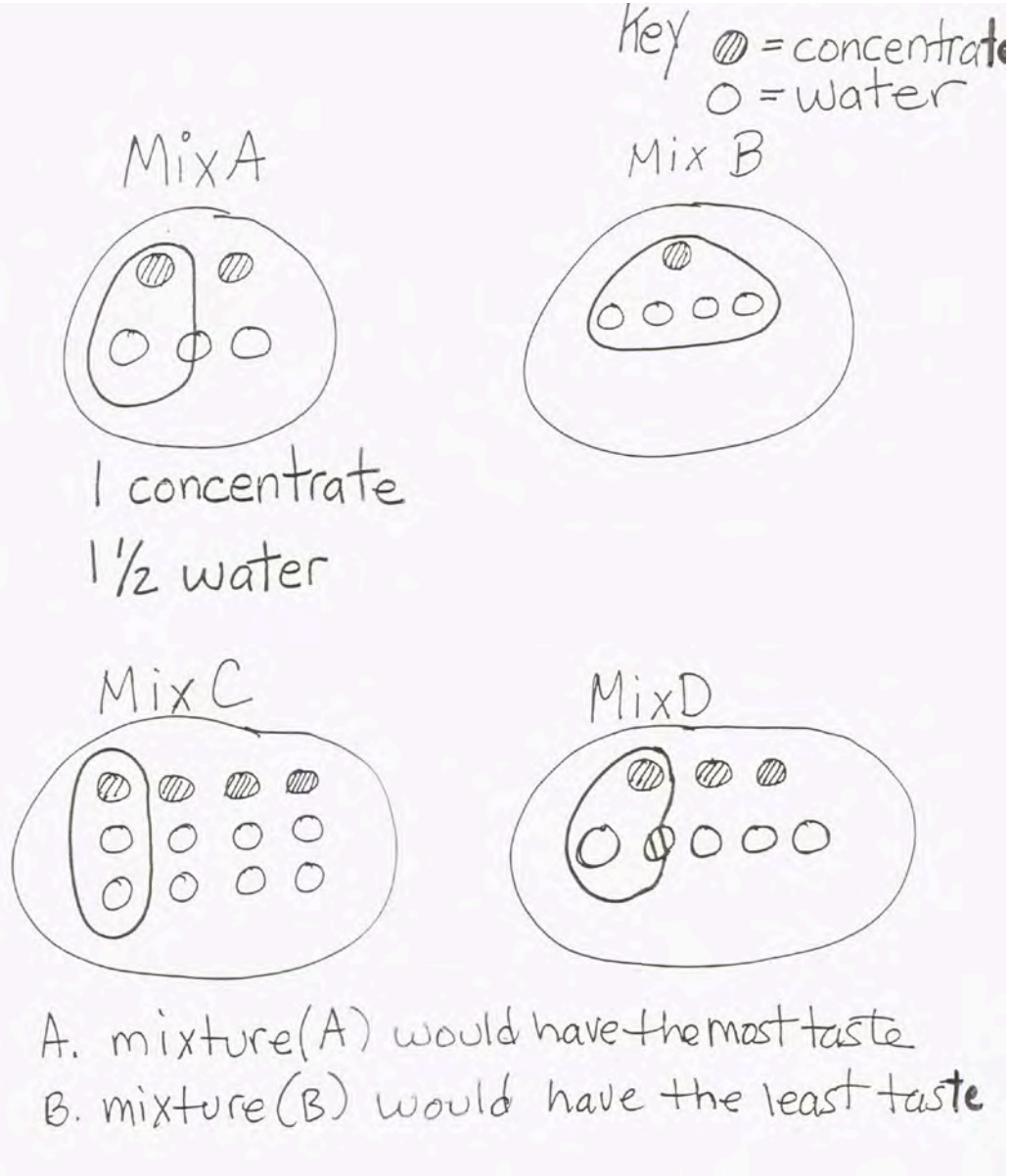
4 cups × 8% = 32% concentrate

Mix D is neither because...

100% ÷ 8 = 12.5% 1 cup = 12.5%

3 × 12.5% = 37.5% concentrate

Other Solutions



Where are the MPs in the task?

- Can you identify places within the lesson that would provide opportunity for students to engage in the Mathematical Practices?

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Proportion task

Mixing Juice

2. Assume that each camper will get ½ cup of juice.
   a) For each mix, how many batches are needed to make juice for 240 campers?
   b) For each mix, how much concentrate and how much water are needed to make juice for 240 campers?

MIX A	
2 cups concentrate	3 cups cold water

MIX B	
5 cups concentrate	9 cups cold water

MIX C	
1 cups concentrate	2 cups cold water

MIX D	
3 cups concentrate	5 cups cold water

Something else to consider…

- Two friends mix blue tint with white paint to make some blue paint. Decide which friend mixed the darkest shade of blue paint. Nancy used more blue tint than Kathy. Nancy mixed in more white paint than Kathy. Who mixed the darkest shade of blue?

Scaffolding an investigation

"Where should they meet?"

Where Should They Meet?

- Five police officers are on the same highway and find that they need to get together for a meeting. They are currently at Mile Markers 2, 10, 11, 37, and 40. They can meet anyplace along the highway.
- Where would you predict they should meet to minimize the miles driven?
- Develop a systematic method for investigating where the meeting should occur to minimize the total number of miles driven. Where should the meeting occur? Justify your results.
- Create a different set of five numbers and investigate where the meeting should occur to minimize the total amount of travel.

Where Should They Meet?

- Compute the mean and median for the values provided in the original problem and in your own set of five values. How does the best place for the meeting compare with these measures? Do you think this will always be the case, no matter what five values are used? Support your answer.
- Where should they meet if, instead of trying to minimize the total number of miles driven, they want to minimize the time it takes to get everyone to the meeting? Explain.

Where Should They Meet?

- Suppose the original group had six officers, with the sixth at Mile Marker 80. Where should the group meet so that the total number of miles traveled is the least possible? Generalize your results for any six numbers. Generalize your results to any set of numbers.
- Consider the following test scores: 0, 80, 88, 92, and 95. In what way might you consider the median to be the best measure of central tendency? In what way might you consider the mean to be the best measure of central tendency?

What about the “By 2”?

- Formative Assessment
  - At every step along the way
  - “Beware the chorus!”
  - Get evidence from all students
- Teacher Reflection
  - Evaluate the data you have
  - Proceed? Remediate? Skip?
- Be Intentional

The Structure

- Engage
  - Check for misconceptions
  - Activate prior knowledge
  - Motivate students to engage in the mathematics
- Explore
  - Student-centered
  - Prior to Explanation
- Explain
  - Students should be contributing, with teacher guidance
- Extend
  - Deepen knowledge; generalize; prove; connect

For More Information

- Visit www.clemson.edu/iim
- Andy Tyminski
  - amt23@clemson.edu
- Bob Horton
  - bhorton@clemson.edu